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COMPREHENSIVE RISK ASSESSMENT BASED ON THE CHOQUET INTEGRAL

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Abstract

This study first introduced the related concept and formula on Choquet integral. Then, the formulas on risk loss, the probability of risk occurrence and the value of each risk are given based on the description of integrated risk assessment problems with interactional risks. Furthermore, the formula of integrated risk assessment based on Choquet integral is shown. Finally, an example is given to show the feasibility and practicability of the proposed method. It is of great significance to consider the assessment regarding the correlation between multiple risks. This paper suggested a new method to calculate the comprehensive risk value according to the role of multiple risk factors. Multiple combined risk values for multiple associated risks are calculated. Therefore, it is a new trial of application of Choqeut Integral method in risk assessment research.

Key Words: Operations Research, Risk, Risk Assessment, Choquet Integral

(Editor's Note: This article does not follow the usual two column format for the journal because of the large number of mathematical formulas)

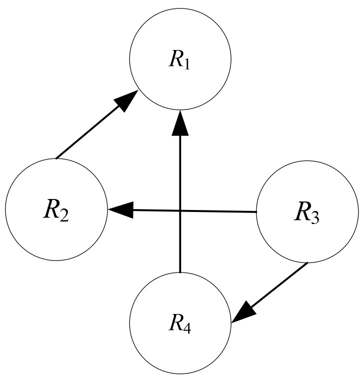
Introduction

Risk assessment is an approach to quantifying the degree of the potential loss or effect of an event or a factor [10]. As an essential element of risk management, risk assessment provides support for risk prevention and control [15]. In recent years, the research on risk assessment has attracted the attention of numerous scholars [4,6,8,10,13, 14,15,16,17,20]. The existing risk assessment methods include Analytic Hierarchy Process (AHP) [16,20], Fuzzy Comprehensive Evaluation [14,17], Monte Carlo Method [4,6,13], Artificial Neural Network [1,3,9], Risk Matrix [7,18], and Program Evaluation and Review Technique (PERT) [2,11]. For example, the AHP method is applied to the evaluation of highway project risks in article [20]; essay [14] presents a comprehensive multi-level fuzzy evaluation model of supply chain risk. The Monte Carlo method is used in article [13] to simulate and evaluate the risk of airport traffic accidents, and the risk prevention measures are provided based on the simulation results. Article [9] proposes an artificial neural network to measure credit risk based on actual cases. Essay [7] puts forward a project risk matrix method, which may not only identify the importance but also assess potential impact of the project risk. In article [5], a PERT method is proposed to measure the risk of virtual enterprises.

The methods as above may resolve the problem of risk assessment from different perspectives, but most of them do not take into account the relationship between multiple risks. In reality, there is, for the most part, a correlation between multiple risks. That is, the occurrence of one risk may affect the occurrence of another, and the non-additivity between different risks should be considered. For instance, as shown in Figure 1, there are some correlations between the five risks (R_1, R_2, R_3, R_4).

Therefore, it is of great significance to consider the assessment regarding the correlation between multiple risks. This paper suggested a new method to calculate the comprehensive risk value according to the role of multiple risk factors. Multiple combined risk values for multiple associated risks are calculated. Therefore, it is a new trial of application of Choqeut Integral method in risk assessment research.

Figure 1. The graphic illustration showing the interrelationship of risks



The Choquet Integral

French mathematician Choquet put forward the theory of capacity in 1954, which was the first systematic study of the non-additive measure. Subsequently, Sugeno, a Japanese scholar, proposed the concept of fuzzy measure to solve the multi-attribute decision-making problem regarding correlation between the nonadditive indicators [12]. The related concepts of the Choquet integral and the corresponding formulas are given below.

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a non-empty set, where x_i represents the i item. μ denotes the function which $P(X)$ projects to $[0,1]$. If the following conditions are satisfied:

1) $\mu(\varnothing) = 0$, $\mu(X) = 1$,
2) $\forall A, B \in P(X)$, $A \subseteq B$,

and $\mu(A) \le \mu(B)$, then μ is considered as the fuzzy measure of X.
If $\forall A, B \in P(X)$, $A \cap B = \varnothing$, and the fuzzy measure μ meets:

$$\mu(A \cup B) = \mu(A) + \mu(B) + \lambda\mu(A)\mu(B), \quad \lambda \in (-1, \infty) \tag{1}$$

then, $\mu(A)$ and $\mu(B)$ denote the fuzzy measure λ of the sets A and B, respectively [19].

In the above equation, $\lambda = 0$ represents the additivity of $\mu(A)$ and $\mu(B)$, indicating that A and B are uncorrelated; $\lambda \ne 0$ signifies that $\mu(A)$ and $\mu(B)$ are non-additive, reflecting the correlation between A and B. Specifically, when $\lambda > 0$, then $\mu(A \cup B) > \mu(A) + \mu(B)$, suggesting that A and B have the multiplication effect, and that μ is superaddicative; when $\lambda < 0$, then $\mu(A \cup B) < \mu(A) + \mu(B)$, indicating the substitution effect of A and B, and showing that μ is subadditive.

The Choquet integral may be applied to the aggregation operation of multiple factors (such as indicators and risks) which are correlated to one another. If f is the nonnegative real valued function of X and μ is the fuzzy measure of X, then the Choquet integral aggregation equation of f and μ may be expressed as:

$$CI_\mu(f) = (C)\int f d\mu \tag{2}$$

Furthermore, the discrete Choquet integral aggregation equation of f and μ may be written as:

$$CI_\mu(f) = \sum_{i=1}^{n} [f(x_{(i)}) - f(x_{(i-1)})]\mu(A_{(i)}) \tag{3}$$

where $(\square)$ represents the permutation of f and X, so that when $f(x_{(1)}) \le f(x_{(2)}) \le \cdots \le f(x_{(n-1)}) \le f(x_{(n)})$, $A_{(i)} = \{x_{(i)}, \cdots, x_{(n)}\}$, $x_{(0)} = 0$.

The Comprehensive Risk Assessment Method

In terms of a comprehensive assessment of multiple risks which are correlated to one another, let the comprehensive risk R consist of $R_1, R_2, \cdots, R_n$. R_i represents the i risk, and $i = 1, 2, \cdots, n$. The set of experts involved in the risk analysis is $E = \{E_1, E_2, \cdots, E_m\}$, where E_k denotes the k expert, and $k = 1, 2, \cdots, m$. Let $w = (w_1, w_2, \cdots, w_m)$ be the weight vector of experts, where w_k signifies the degree of importance or weight of the expert, and it satisfies $\sum_{k=1}^{m} w_k = 1$, $0 \le w_k \le 1$, $k = 1, 2, \cdots, m$. Let $L_k = (l_1^k, l_2^k, \cdots, l_n^k)$ be the risk loss vector proposed by expert E_k, where l_i^k represents the loss due to the risk predicted by the expert E_k. Let $P_k = (p_1^k, p_2^k, \cdots, p_n^k)$ be the risk probability vector presented by expert E_k, where p_i^k indicates the probability of risk R_i predicted by the expert E_k. The risk loss l_i^k and the probability of risk occurrence p_i^k may be obtained through a statistical analysis of historical data or through the experts' experiences and subjective judgments.

More precisely, let $\mu_k = (\mu_{1\square n}^k, \mu_{2\square n}^k, \cdots, \mu_{n\square n}^k)$ be the fuzzy measure vector given by the expert E_k, where $\mu_{i\square n}^k$ is the fuzzy measure of the risk set $\{R_i, R_{i+1}, \cdots, R_n\}$ predicted by the expert E_k. That is, after considering the correlation of the risks in the set $\{R_i, R_{i+1}, \cdots, R_n\}$, the expert E_k gives the degree of importance of $\{R_i, R_{i+1}, \cdots, R_n\}$ in the whole risk set. Let $Z = (z_1, z_2, \cdots, z_n)$ be the risk value vector, where z_i represents the value of the risk R_i;let z_s denotes the value of the comprehensive risk R.

The problem to be resolved is: considering the correlation between multiple risks, the paper, based on the decision-making information provided by the expert group (namely, the risk loss vector L_k, the risk probability vector P_k, and the fuzzy measure vector μ_k), uses the decision analysis method to calculate the value of the comprehensive risk R. Since there is a correlation between multiple risks, the value of the overall risk R cannot be simply summed by the value of the risk $R_1, R_2, \cdots, R_n$. Therefore, the paper uses the Choquet integral to calculate the aggregation of the multiple risk values.

First, m risk-loss vectors $L_1, L_2, \cdots, L_m$ are aggregated into a group risk loss vector $L_G = (l_1^G, l_2^G, \cdots, l_n^G)$, which is calculated as:

$$l_i^G = \sum_{k=1}^{m} w_k l_i^k, \quad i = 1, 2, \cdots, n \quad , \tag{4}$$

m risk probability vectors $P_1, P_2, \cdots, P_m$ are aggregated into a group risk probability vector $P_G = (p_1^G, p_2^G, \cdots, p_n^G)$, which is written as:

$$p_i^G = \sum_{k=1}^{m} w_k p_i^k, \quad i = 1, 2, \cdots, n \tag{5}$$

Then, according to the group risk loss vector L_G and the group risk probability vector P_G, the paper calculates the risk value vector $Z = (z_1, z_2, \cdots, z_n)$, and the equation of z_i is:

$$z_i = l_i^G p_i^G, \quad i = 1, 2, \cdots, n \tag{6}$$

Furthermore, according to the risk value $z_1, z_2, \cdots, z_n$, the n risks $R_1, R_2, \cdots, R_n$ are rearranged in ascending order, so that the relationship between the values $z_1', z_2', \cdots, z_n'$ of the risk $R_1', R_2', \cdots, R_n'$ meets $z_1' \le z_2' \le \cdots \le z_{n-1}' \le z_n'$. Meanwhile, m fuzzy measure vectors $\mu_1, \mu_2, \cdots, \mu_m$ are aggregated into a group fuzzy measure vector $\mu_G = (\mu_{1\square n}^{G}, \mu_{2\square n}^{G}, \cdots, \mu_{n\square n}^{G})$, where

$$\mu_{i\square n}^{G} = \sum_{k=1}^{m} w_k \mu_{i\square n}^{k}, \quad i = 1, 2, \cdots, n \tag{7}$$

Finally, according to equation (3), the paper calculates the comprehensive risk z_s:

$$z_s = \sum_{i=1}^{n} (z_i' - z_{i-1}') \mu_{i\square n}^{G} \tag{8}$$

where $z_0' = 0$, z_s is the value of the comprehensive risk which is obtained through an aggregation of n correlated risks. Managers may prevent and control risks according to z_s.

Overall, the steps of the comprehensive risk assessment method based on Choquet integral are as follows:

Step 1. Calculate the group risk loss vector L_G according to Eq. (4);

Step 2. Calculate the group risk probability vector P_G according to Eq. (5);

Step 3. Calculate the risk value vector Z according to Eq. (6);

Step 4. Calculate the group fuzzy measure vector μ_G according to Eq. (7);

Step 5. Calculate the comprehensive risk value according to Eq. (8).

A Case Study

As Steel Company in North China wants to outsource its logistics business to a local logistics outsourcing company. But AS needs to assess the potential risks of outsourcing activity first. In order to assess the risk of outsourcing, AS invites five experts to form a risk assessment team, and then each expert, according to their experiences and professional skills, provides the weight vector $w = (0.3, 0.2, 0.1, 0.3, 0.1)$. Through a systematic analysis of the related research and the feedback of the questionnaires, the expert team determines that the logistics outsourcing risk consists of four potential risks: hidden transaction cost R_1, contract dispute R_2, decline in the service quality R_3, revelation of trade secrets R_4. Based on historical data and professional experiences, the five experts provide four kinds of risk losses (in a unit of ten thousand yuan) and four risk probabilities, as shown in Table 1.

To resolve the multiple risk assessment problem mentioned above, a brief description of the calculation process is presented as follows: First, the paper calculates the group risk loss vector $L_G = (5.9, 4.4, 4.6, 11.2)$ according to Eq. (4). Second, the group risk probability vector $P_G = (0.339, 0.248, 0.437, 0.140)$ is obtained according to Eq. (5). Third, the paper calculates the risk value vector $Z = (2.0001, 1.0912, 2.0102, 1.568)$ according to Eq. (6). According to the obtained risk vector Z, the four risks are

Table 1. The risk loss vectors and risk probability vectors given by experts

E_k	L_k	P_k
E_1	(5, 4, 3, 10)	(0.36, 0.28, 0.42, 0.12)
E_2	(6, 3, 5, 13)	(0.30, 0.25, 0.48, 0.15)
E_3	(4, 5, 7, 9)	(0.35, 0.20, 0.40, 0.16)
E_4	(7, 5, 5, 12)	(0.32, 0.24, 0.45, 0.14)
E_5	(7, 6, 5, 11)	(0.4, 0.22, 0.40, 0.16)

rearranged in ascending order: contract dispute R_1', trade secret disclosure R_2', hidden transaction cost R_3', and decline in the service quality R_4'. On this basis, five experts provide the corresponding fuzzy measure vector for $\{R_4'\}$, $\{R_3', R_4'\}$, and $\{R_1', R_2', R_3', R_4'\}$, as shown in Table 2. Then, the paper proceeds to calculate the group fuzzy measure vector $\mu_G = (1, 0.859, 0.53, 0.332)$ according to Eq. (7). Eventually, the comprehensive risk $z_s = \sum_{i=1}^{4} (z_i' - z_{i-1}') \mu_{i\square 4}^G$ =1.733 is obtained according to Eq. (8).

Table 2. Fuzzy measure vectors given by experts

E_k	$\mu_k = (\mu_{1\square 4}^k, \mu_{2\square 4}^k, \mu_{3\square 4}^k, \mu_{4\square 4}^k)$			
	$\mu_{1\square 4}^k$	$\mu_{2\square 4}^k$	$\mu_{3\square 4}^k$	$\mu_{4\square 4}^k$
E_1	1	0.88	0.56	0.3
E_2	1	0.8	0.5	0.36
E_3	1	0.8	0.6	0.4
E_4	1	0.9	0.5	0.3
E_5	1	0.85	0.52	0.4

Conclusions

Under the condition of a given risk loss and risk probability, this paper uses the Choquet integral method to address the assessment of comprehensive risks which are related but non- additive. The case study shows the applicability and effectiveness of the Choquet integral method which is featured by its clear concept and simple calculation. Nevertheless, to resolve the problem of the complicated evaluation process and heavy computational burden caused by multiple risks to the expert group, further studies should be conducted.

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